

TABLE 1 Combinations and Permutations With and Without Repetition.

Type	Repetition Allowed?	Formula
r -permutations	No	$\frac{n!}{(n-r)!}$
r -combinations	No	$\frac{n!}{r!(n-r)!}$
r -permutations	Yes	n^r
r -combinations	Yes	$\frac{(n+r-1)!}{r!(n-1)!}$

PRINCIPLE OF MATHEMATICAL INDUCTION To prove that $P(n)$ is true for all positive integers n , where $P(n)$ is a propositional function, we complete two steps:

BASIS STEP: We verify that $P(1)$ is true.

INDUCTIVE STEP: We show that the conditional statement $P(k) \rightarrow P(k+1)$ is true for all positive integers k .

Template for Proofs by Mathematical Induction

1. Express the statement that is to be proved in the form “for all $n \geq b$, $P(n)$ ” for a fixed integer b .
2. Write out the words “Basis Step.” Then show that $P(b)$ is true, taking care that the correct value of b is used. This completes the first part of the proof.
3. Write out the words “Inductive Step.”
4. State, and clearly identify, the inductive hypothesis, in the form “assume that $P(k)$ is true for an arbitrary fixed integer $k \geq b$.”
5. State what needs to be proved under the assumption that the inductive hypothesis is true. That is, write out what $P(k+1)$ says.
6. Prove the statement $P(k+1)$ making use of the assumption $P(k)$. Be sure that your proof is valid for all integers k with $k \geq b$, taking care that the proof works for small values of k , including $k = b$.
7. Clearly identify the conclusion of the inductive step, such as by saying “this completes the inductive step.”
8. After completing the basis step and the inductive step, state the conclusion, namely that by mathematical induction, $P(n)$ is true for all integers n with $n \geq b$.

STRONG INDUCTION To prove that $P(n)$ is true for all positive integers n , where $P(n)$ is a propositional function, we complete two steps:

BASIS STEP: We verify that the proposition $P(1)$ is true.

INDUCTIVE STEP: We show that the conditional statement $[P(1) \wedge P(2) \wedge \dots \wedge P(k)] \rightarrow P(k+1)$ is true for all positive integers k .

The events E_1, E_2, \dots, E_n are *pairwise independent* if and only if $p(E_i \cap E_j) = p(E_i)p(E_j)$ for all pairs of integers i and j with $1 \leq i < j \leq n$. These events are *mutually independent* if $p(E_{i_1} \cap E_{i_2} \cap \dots \cap E_{i_m}) = p(E_{i_1})p(E_{i_2}) \dots p(E_{i_m})$ whenever $i_j, j = 1, 2, \dots, m$, are integers with $1 \leq i_1 < i_2 < \dots < i_m \leq n$ and $m \geq 2$.

Let E and F be events with $p(F) > 0$. The *conditional probability* of E given F , denoted by $p(E | F)$, is defined as

$$p(E | F) = \frac{p(E \cap F)}{p(F)}.$$

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If n and r are integers with $0 \leq r \leq n$, then $P(n, r) = \frac{n!}{(n-r)!}$.

The probability of exactly k successes in n independent Bernoulli trials, with probability of success p and probability of failure $q = 1 - p$, is

$$C(n, k)p^k q^{n-k}.$$